Transverse quark densities

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Outline

What do we know about the transverse structure (imaging) of hadrons ?

- Light-front charge densities <-> elastic nucleon Form Factors
- Shape of hadrons <-> higher e.m. moments of transverse charge densities
  (systems of spin 1 or higher)
- Electromagnetic structure of the Δ(1232) resonance

  transverse charge densities from recent lattice results
- Baryon resonance structure / transition charge densities
  <-> N -> N^* Transition Form Factors

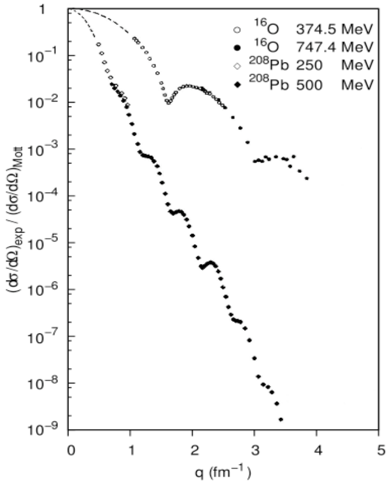
  Δ(1232), P_{11}(1440), S_{11}(1535), D_{13}(1520),...

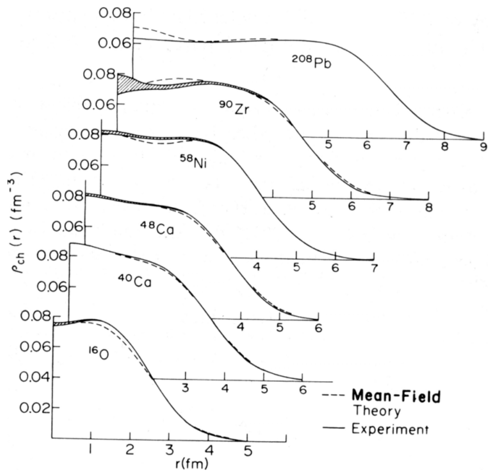
size and shape of non-relativistic many-body systems

Sizes of nuclei

as revealed through

elastic electron scattering

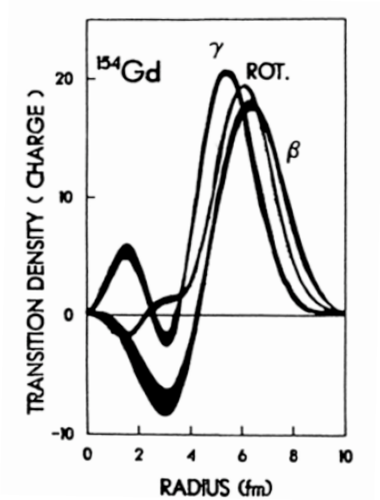


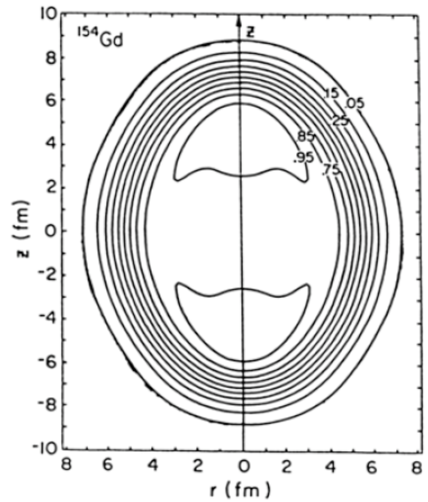


Shapes of deformed nuclei

as revealed through

inelastic electron scattering





perspective on "Shape of Hadrons" : Alexandrou, Papanicolas, Vdh (2011)

size of proton : electric charge radius

Lamb shift in muonic H (PSI)

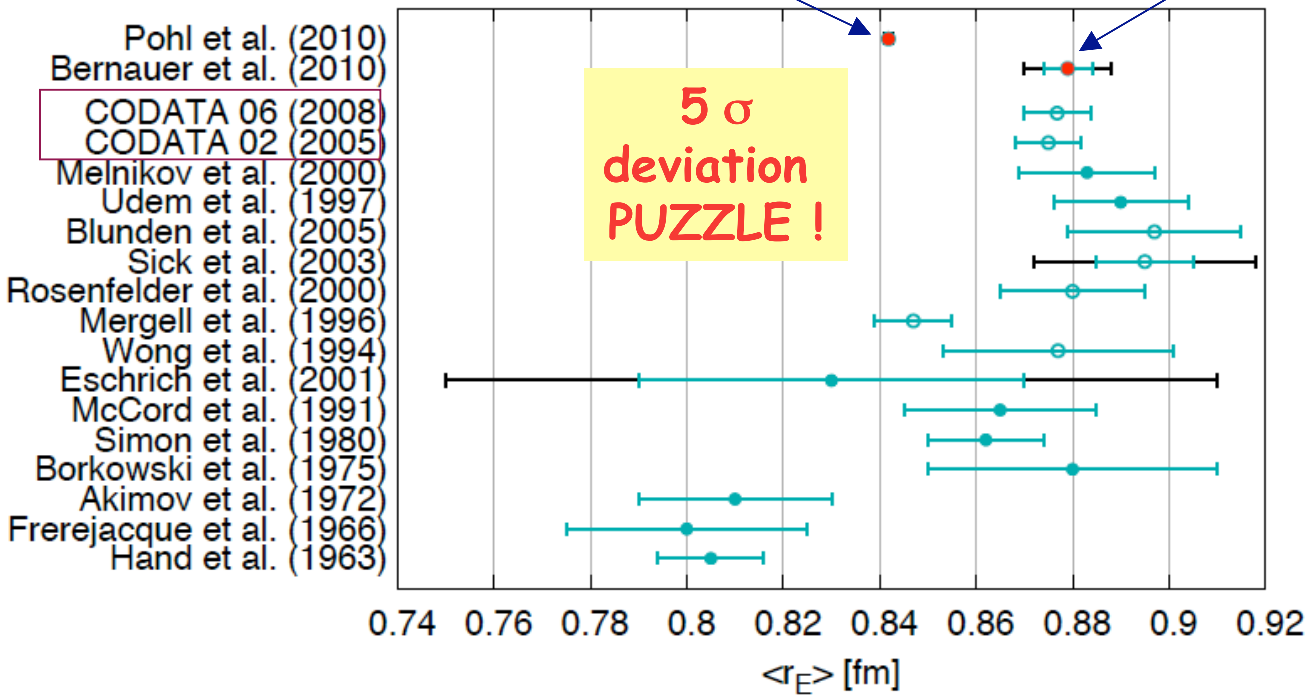
Pohl et al.
Nature 466 (2010) 213

$\langle r_E \rangle$ = 0,84184(67) fm

ep-scattering (MAMI)

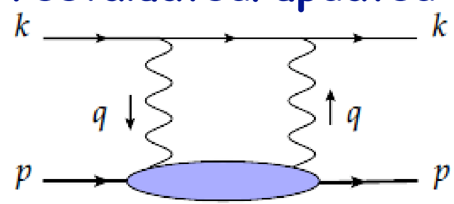
Bernauer et al.
PRL 105 (2010) 242001

$\langle r_E \rangle$ = 0,879(8) fm



corrections to Lamb shift are 300 μeV below expectation

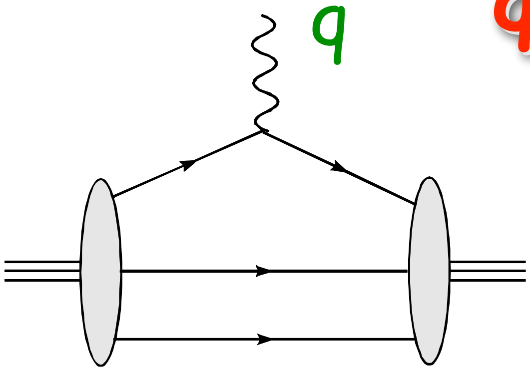
proton structure corrections have been reevaluated/updated

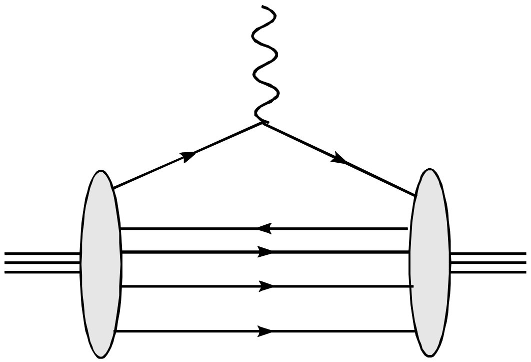


$\Delta E = (-\ 36.9 \pm 2.4)\ \mu eV$

Carlson, Vdh (2011)

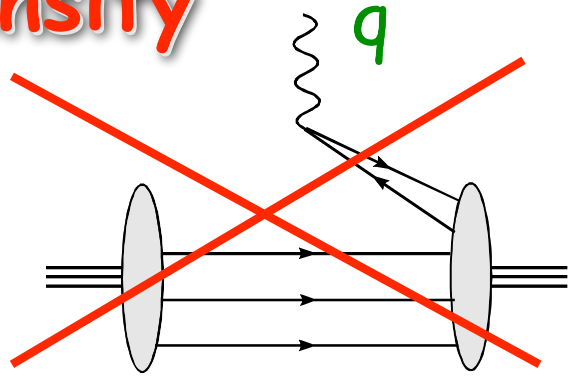
interpretation of Form Factor as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

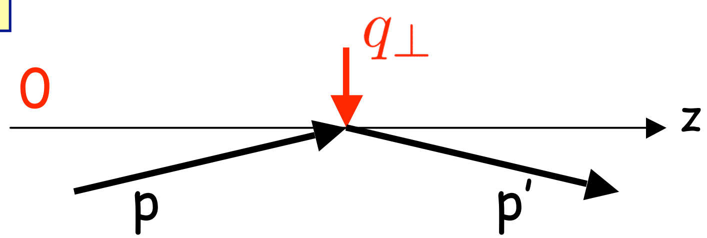
NO probability/charge density interpretation

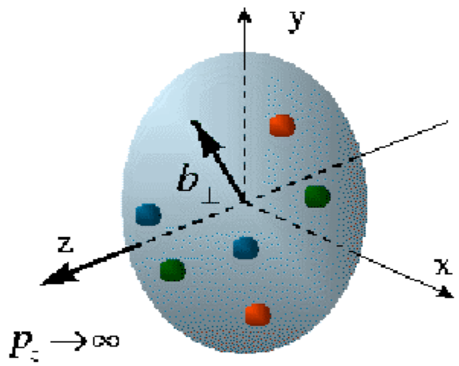
absent in a LIGHT-FRONT frame !

$$q^+ = q^0 + q^3 = 0$$

quark transverse charge densities in nucleon (I)

light-front

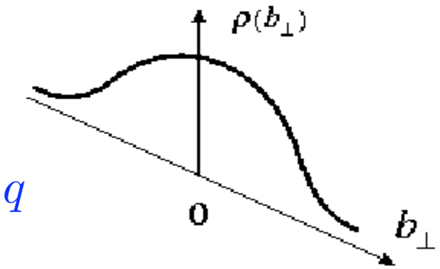




- $q^+ = q^0 + q^3 = 0$

  $Q^2 \equiv \vec{q}_\perp^{\,2}$

  photon only couples to forward moving quarks



- quark charge density operator

$$J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+ q = 2\, q_+^\dagger\, q_+, \quad \text{with} \quad q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$$

☆ longitudinally polarized nucleon

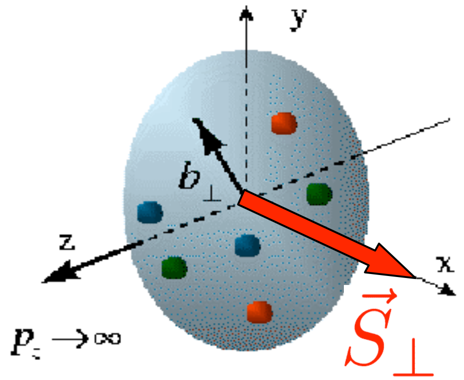
$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda\,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

Miller (2007)

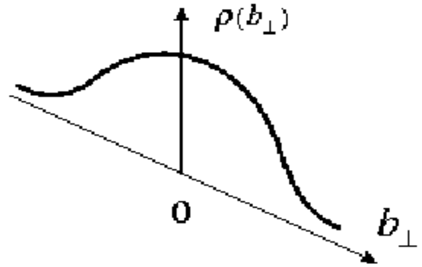
quark transverse charge densities in nucleon (II)

☆ transversely polarized nucleon



transverse spin $\vec{S}_\perp = \cos\phi_S\,\hat{e}_x + \sin\phi_S\,\hat{e}_y$

e.g. along x-axis : $\phi_S = 0$



$$\vec{b} = b\,(\cos\phi_b\,\hat{e}_x + \sin\phi_b\,\hat{e}_y)$$

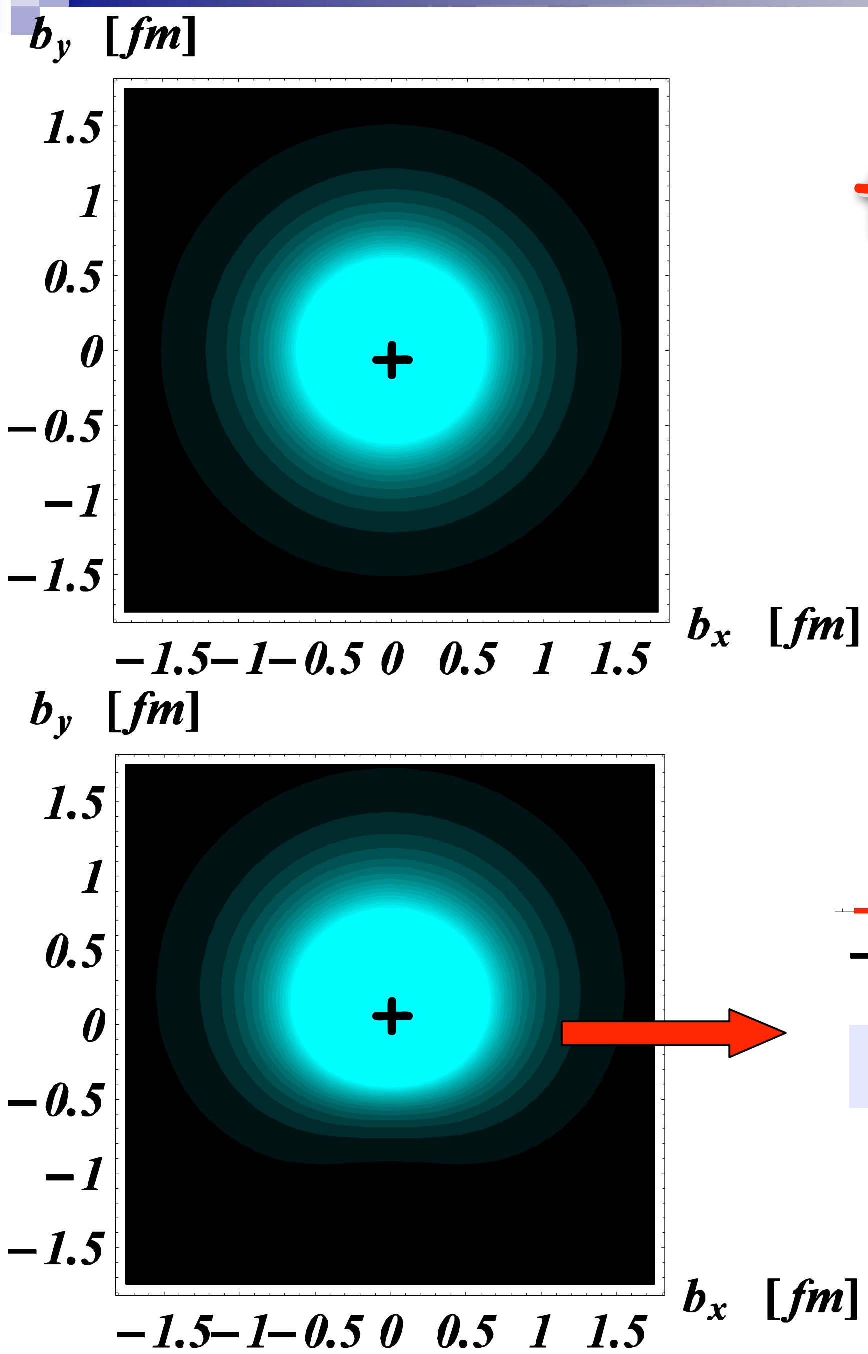
$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\,|\,J^+(0)\,|\,P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\rangle$$

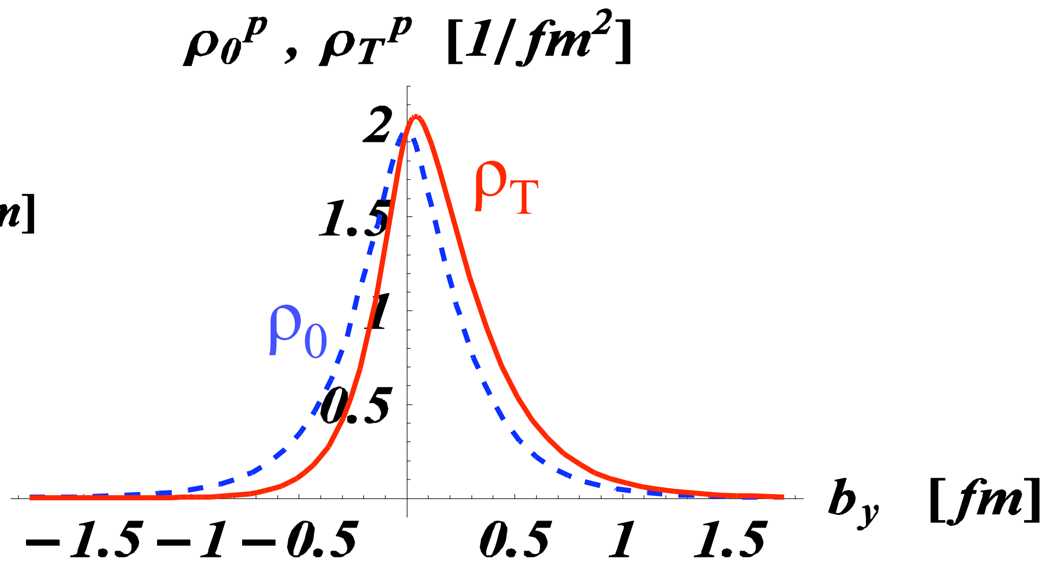
$$= \rho_0^N(b) + \sin(\phi_b - \phi_S)\int_0^\infty \frac{dQ}{2\pi}\,\frac{Q^2}{2M_N}\,J_1(b\,Q)F_2(Q^2)$$

dipole field pattern

Carlson, Vdh (2007)

empirical quark transverse densities in proton





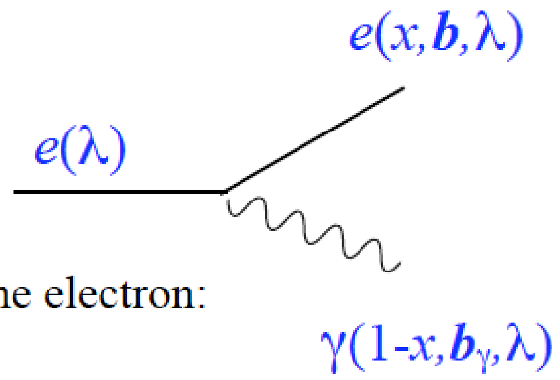
induced EDM : $d_y = F_{2p}(0) \cdot e / (2 M_N)$

data : Arrington, Melnitchouk, Tjon (2007)

densities : Miller (2007) ; Carlson, Vdh (2007)

transverse densities of eγ Fock state in electron

Hoyer, Kurki (2009)



The wave functions give the densities of the |eγ⟩ Fock state of the electron:

$$\rho_0(x,\boldsymbol{b}) = \frac{\alpha m^2}{2\pi^2}\left[\frac{1+x^2}{1-x}K_1^2(mb) + (1-x)\,K_0^2(mb)\right]$$

LC Wavefunction :

Brodsky, Drell (1980)

$$\rho_x(x,\boldsymbol{b}) = \rho_0(x,\boldsymbol{b}) + \frac{\alpha m^2}{\pi^2} x\,\sin(\phi_b)\,K_0(mb)K_1(mb)$$

from which the Pauli form factor is obtained (exact at order α)

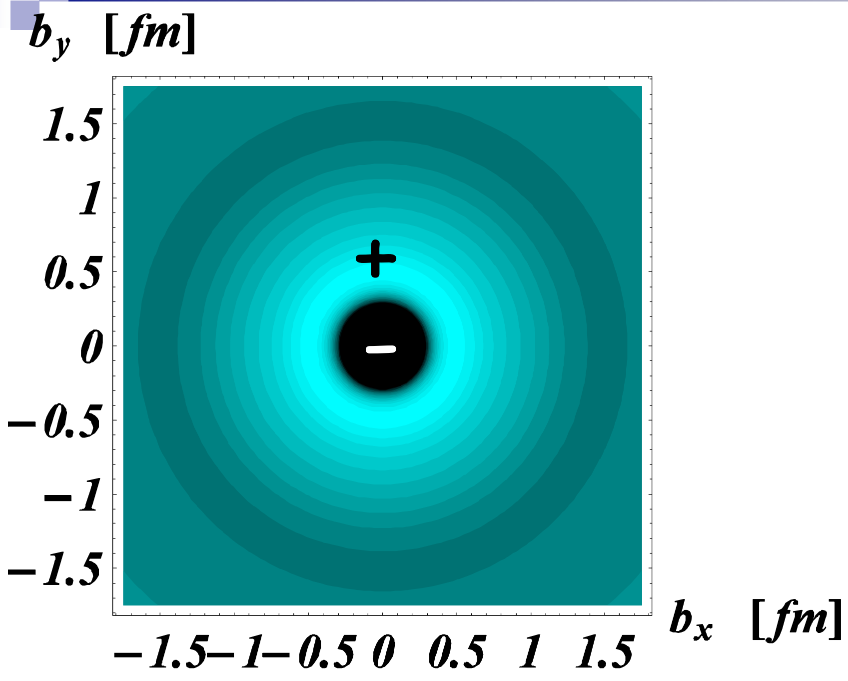
$$F_2(Q^2) = \frac{4\alpha m^3}{\pi Q}\int_0^1 dx\,x\int_0^\infty db\,b\,J_1(b\,Q)K_0(mb)K_1(mb)$$

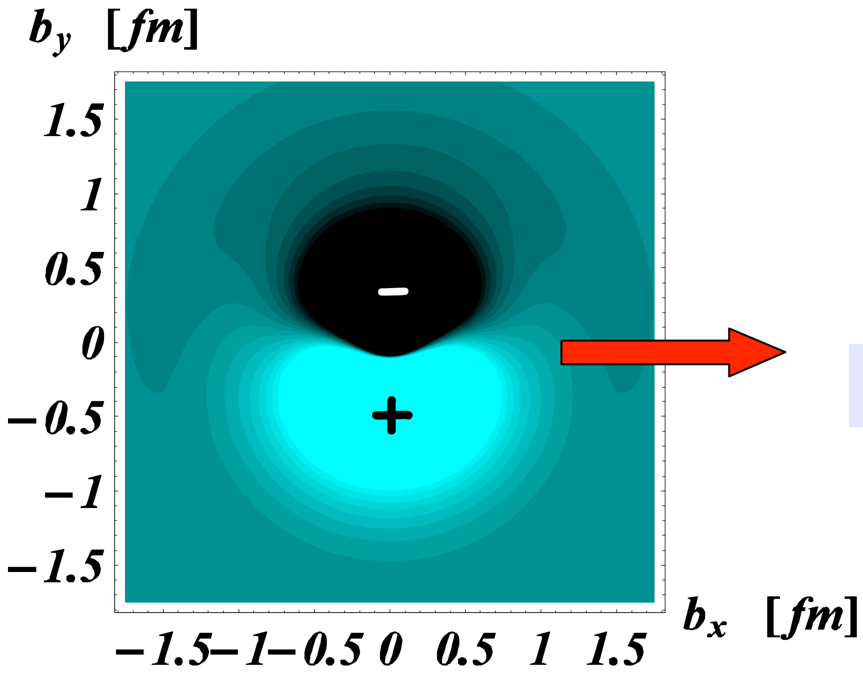
x- and *b*-dependence factorizes

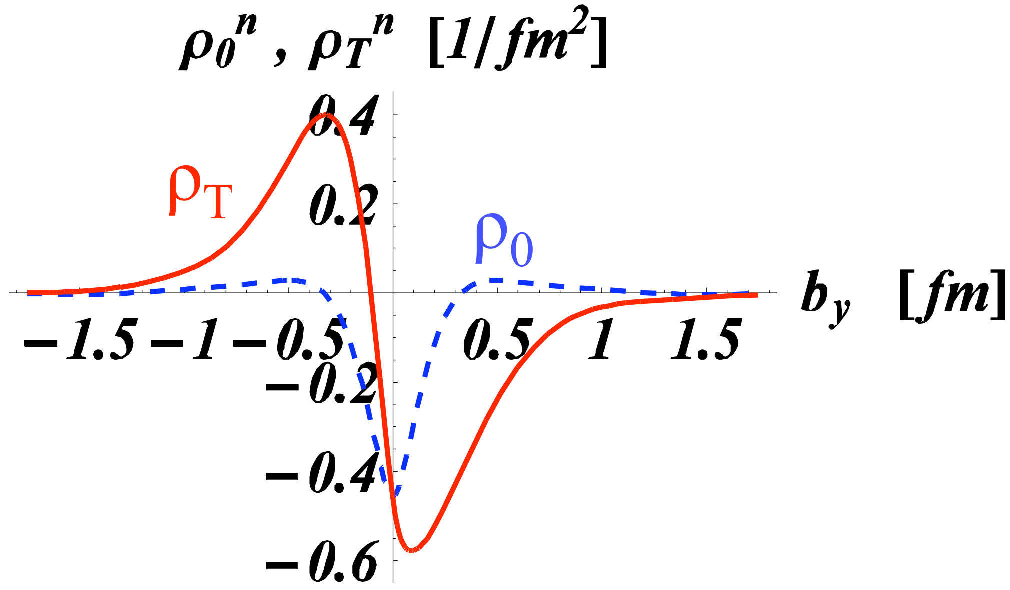
$$= \frac{2\alpha m^2}{\pi}\frac{1}{Q\sqrt{Q^2+4m^2}}\log\left[\frac{1}{2m}\left(\sqrt{Q^2+4m^2}+Q\right)\right]$$

Exact expression from loop integral

empirical quark transverse densities in neutron







induced **EDM** : d_y = F_{2n} (0) . e / (2 M_N)

data: Bradford, Bodek, Budd, Arrington (2006)

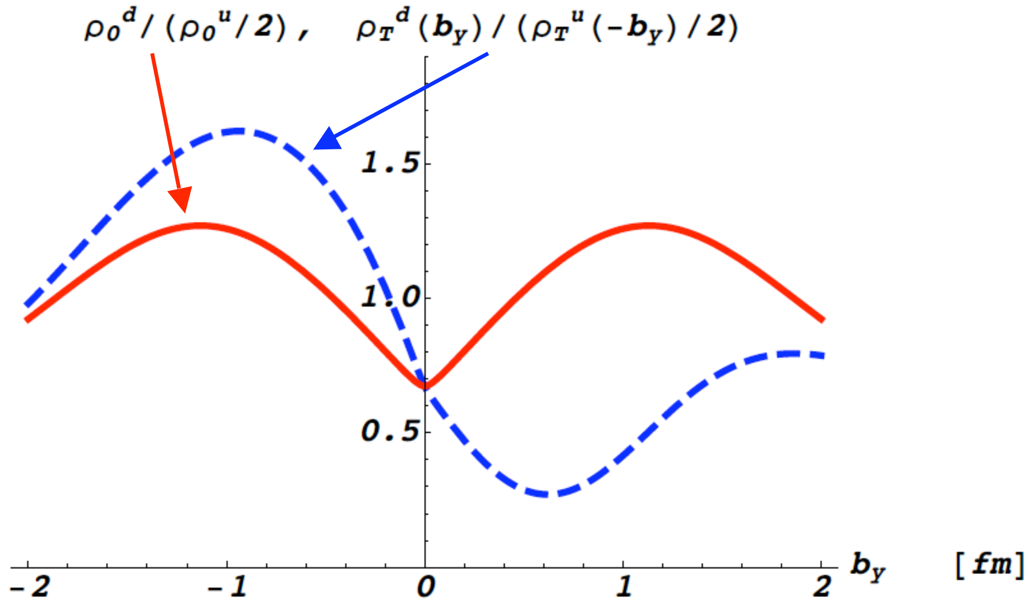
densities : Miller (2007); Carlson, Vdh (2007)

d/u quark spatial distributions

2D spatial distr. :

d-quark distr. spread out further in proton compared to u-quark distr.

Opposite behavior for neutron



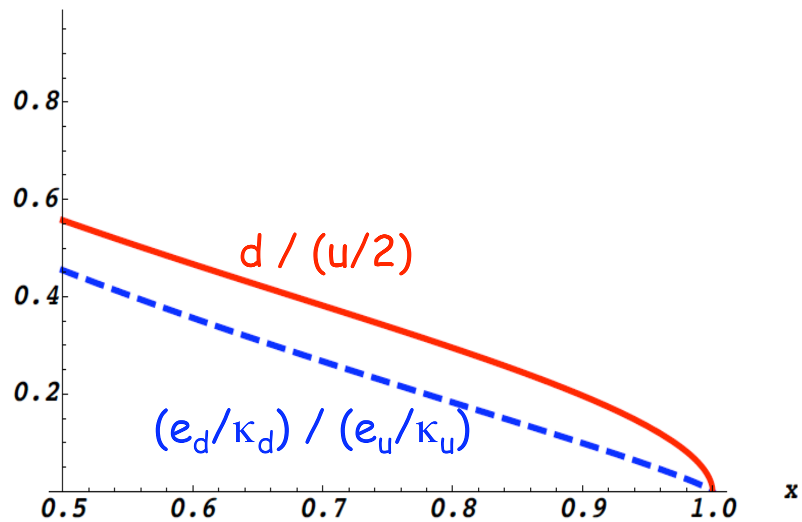
d/u quark momentum distributions

transverse spatial and

longitudinal momentum distribution

can be combined in a 3D picture of

nucleon, described through a GPD



Generalized Parton Distributions (GPDs) : 3D picture of nucleon

DIS
longitudinal quark distribution in momentum space

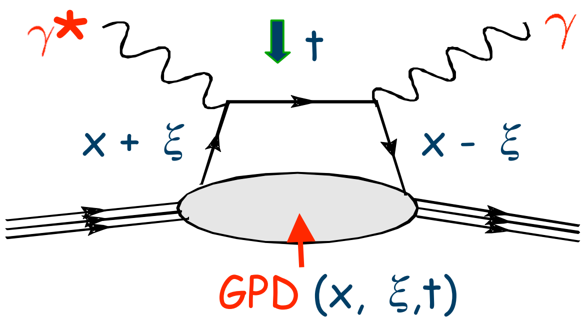
Unifying concept

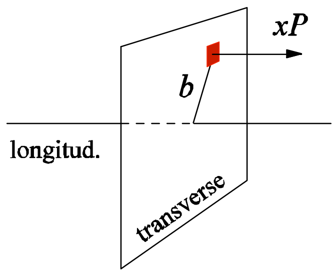
GPDs

Elastic Scattering
transverse quark distribution in coordinate space

fully-correlated quark distributions in **both** coordinate and momentum space

$Q^2 >> 1\ GeV^2$





Burkardt (2000, 2003), Belitsky, Ji, Yuan (2004)

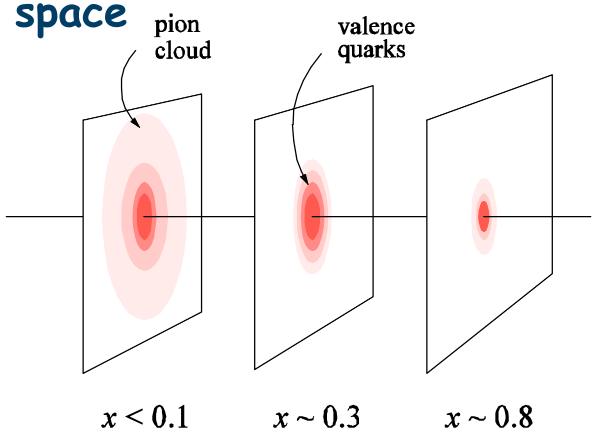


figure courtesy **Weiss**

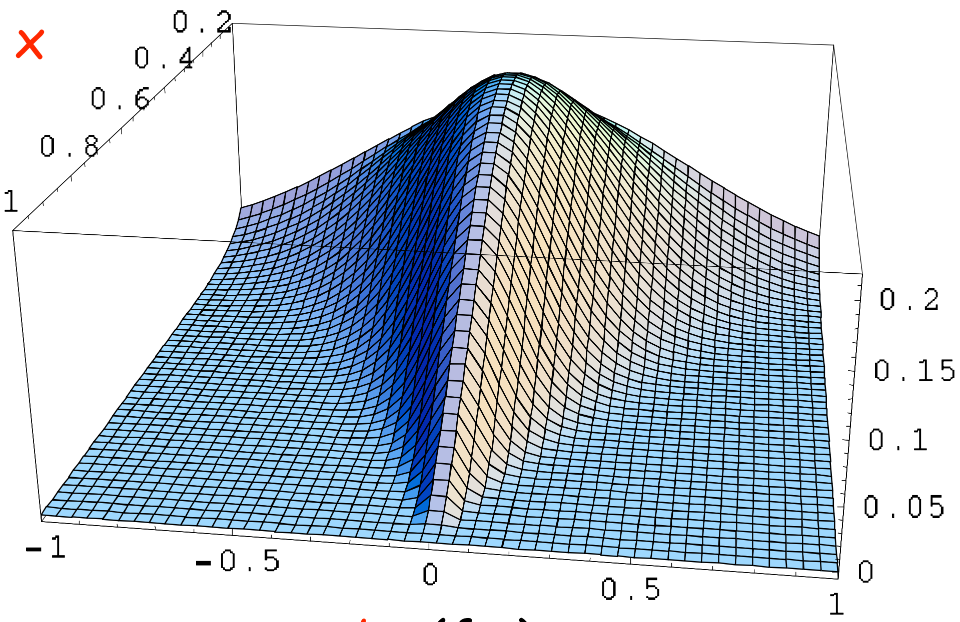
GPDs : transverse image of nucleon

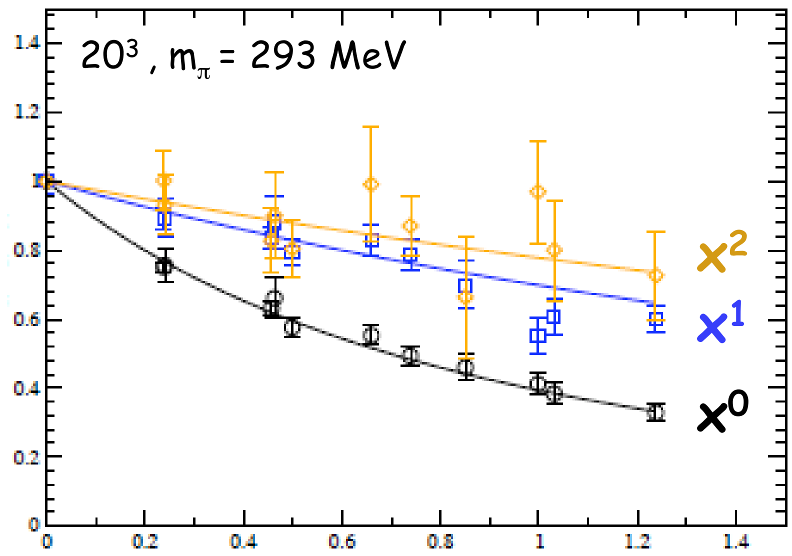
GPDs : quark distributions w.r.t. longitudinal momentum x and transverse position $b_\perp$

lattice QCD : moments of GPDs

$H^u(x, b_\perp)$

x^n moment of H^{u-d}





$b_\perp$ (fm)

Fourier transform

$-t$ (GeV^2)

LHPC Coll.

Guidal, Polyakov, Radyushkin, Vdh (2005),

Diehl, Feldmann, Jakob, Kroll (2005)

Spin-1 transverse densities

transversely polarized deuteron

$$Q^d_{s_\perp} \equiv e \int d^2\vec{b}\,(b_x^2 - b_y^2)\,\rho^d_{T\,s_\perp}(\vec{b})$$

$$Q^d_1 = -\frac{1}{2}Q^d_0 = \frac{1}{2}\{[G_M(0) - 2] + [G_Q(0) + 1]\}\left(\frac{e}{M_d^2}\right)$$

experiment :

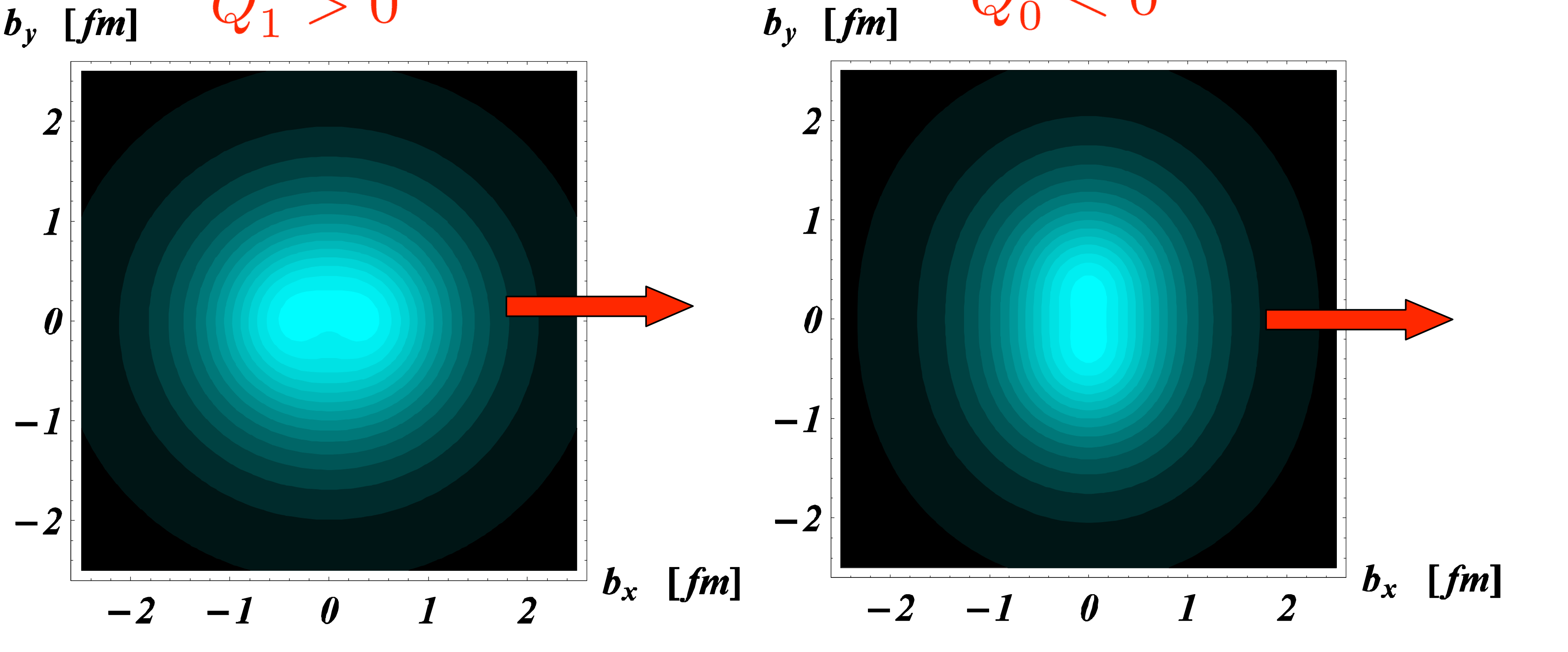
$$G_M(0) = 1.71$$

$$G_Q(0) = 25.84(3)$$

$s_\perp = +1$ $\qquad$ $s_\perp = 0$

Carlson, Vdh (2008)

$Q^d_1 > 0$ $\qquad$ $Q^d_0 < 0$



E.M. moments of W bosons

for spin-1 point particle

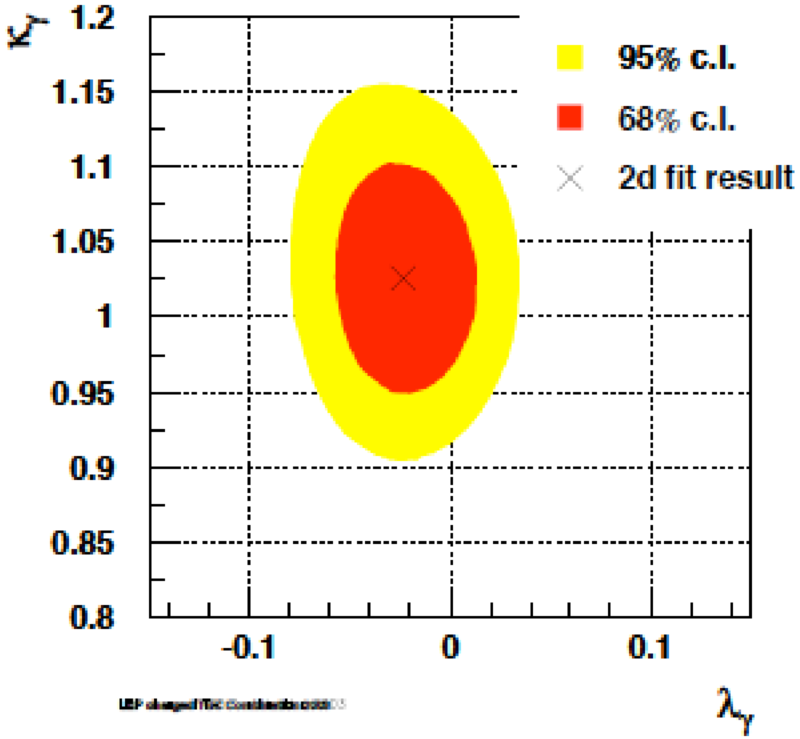
$G_M(0) = 2$ and $G_Q(0) = -1$

$$\mu = \frac{e}{2M_W}\{2 + (\kappa - 1) + \lambda\}$$

$$Q = -\frac{e}{M_W^2}\{1 + (\kappa - 1) - \lambda\}$$

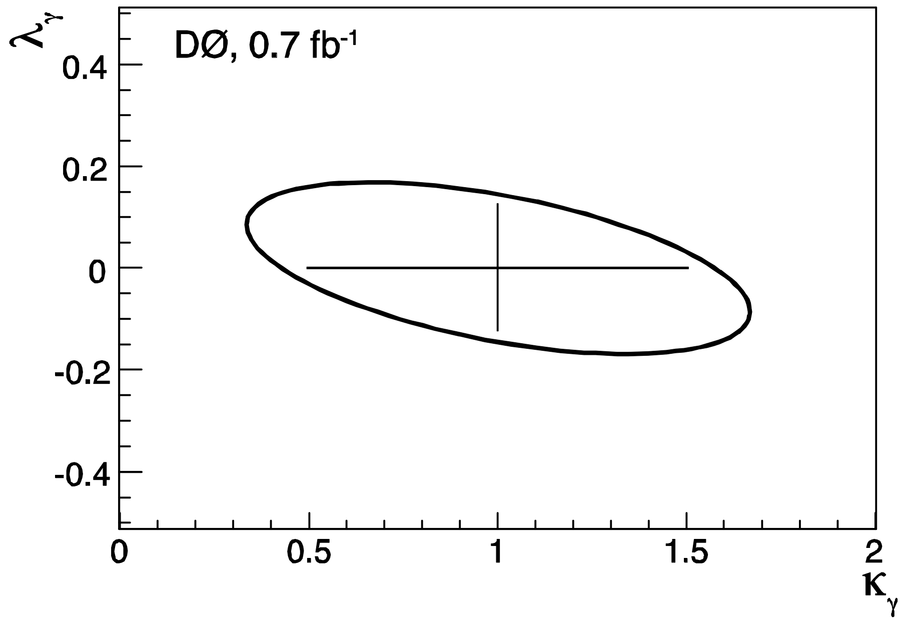
LEP Electroweak working group

hep-ex/0612034



DO Collaboration

PRL100, 241805 (2008)



natural values for e.m. moments of point particle with spin j

Lorcé (2008)

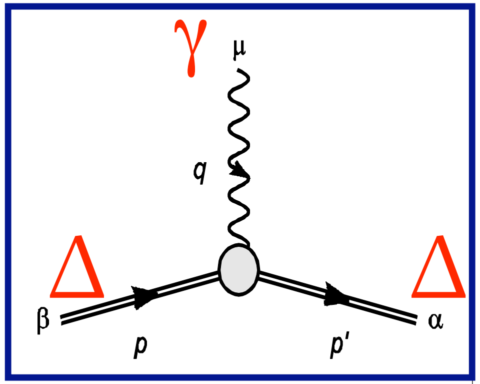
$$
\begin{aligned}
G_{E0}(0) &= 1 \\
G_{M1}(0) &= 2j \\
G_{E2}(0) &= -j(2j-1) \\
G_{M3}(0) &= -\frac{1}{3}j(2j-1)(2j-2)
\end{aligned}
$$

j	$G_{E0}(0)$	$G_{M1}(0)$	$G_{E2}(0)$	$G_{M3}(0)$	$G_{E4}(0)$	$G_{M5}(0)$	$G_{E6}(0)$
0	1	0	0	0	0	0	0
1/2	1	1	0	0	0	0	0
1	1	2	-1	0	0	0	0
3/2	1	3	-3	-1	0	0	0
2	1	4	-6	-4	1	0	0
5/2	1	5	-10	-10	5	1	0
3	1	6	-15	-20	15	6	-1
...							

→ transverse charge densities depend only on anomalous values of e.m. moments → reflect internal structure

Spin-3/2 transverse densities

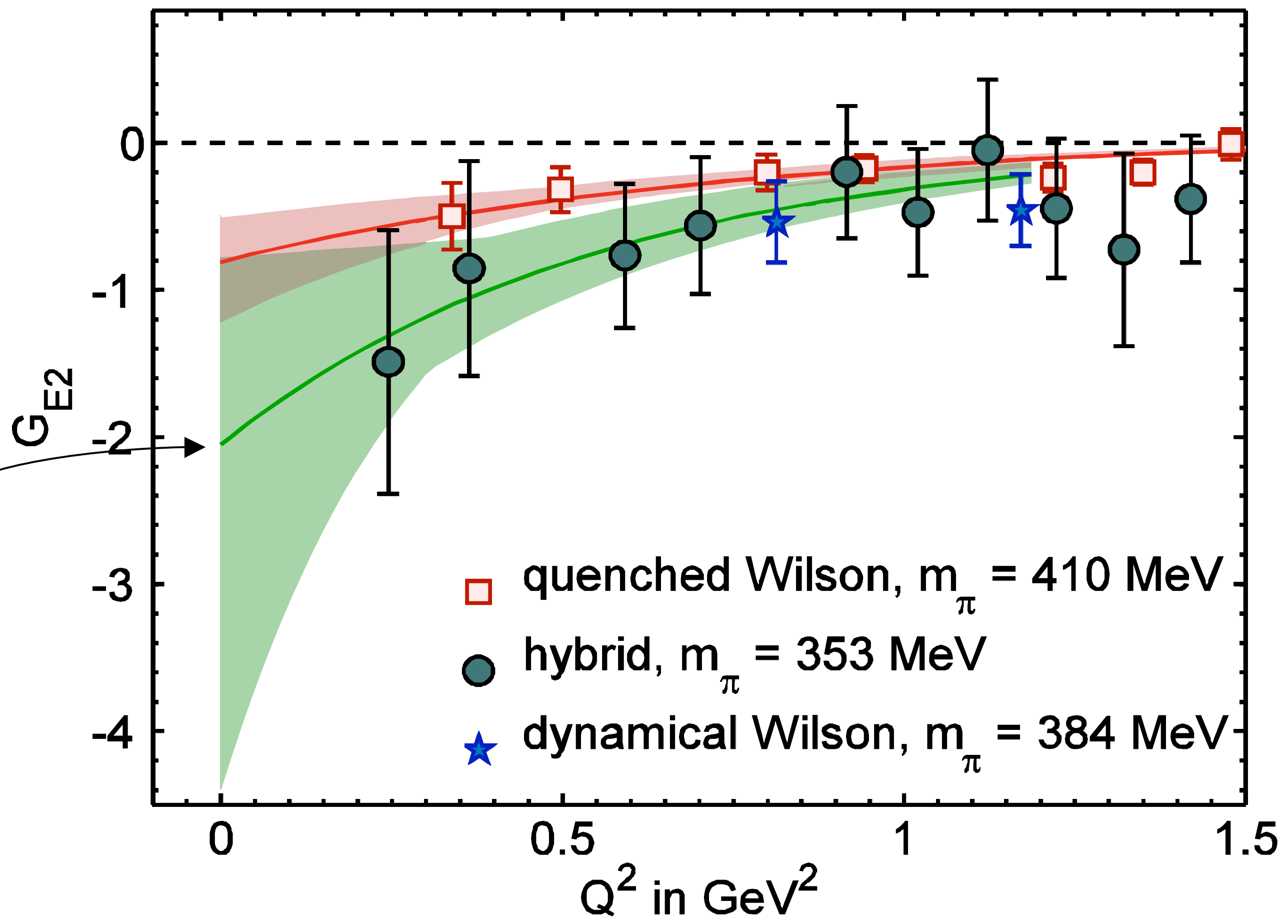
Hadron shape : e.m. Δ to Δ transition



C0 , M1 ,

C2 , M3

transitions



lattice analysis :

Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)

quark transverse charge densities in $\Delta(1232)$

$$\rho^{\Delta}_{T\,s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \rangle$$

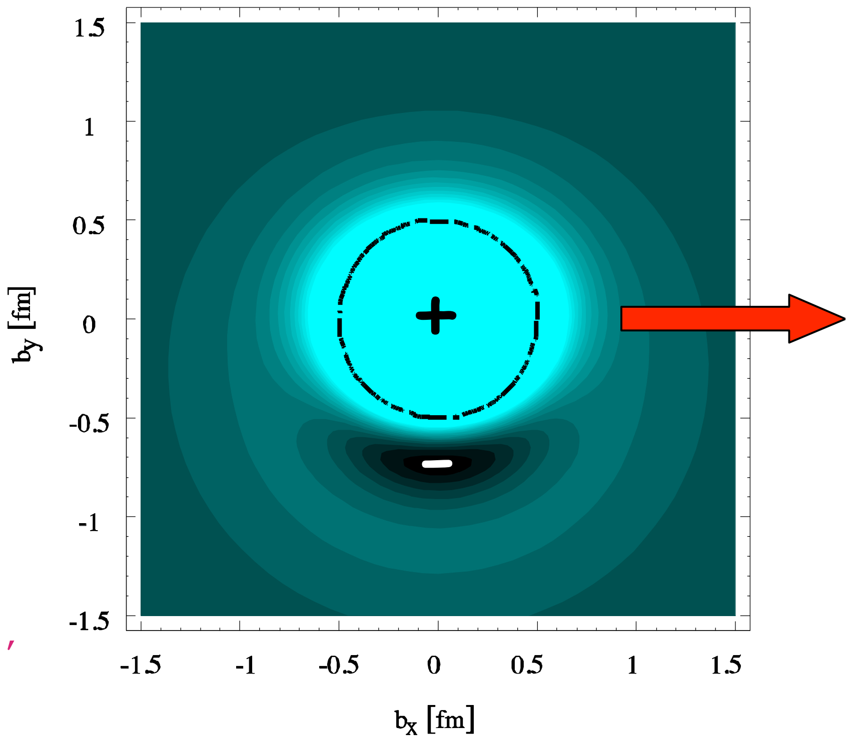
$$Q^{\Delta}_{s_\perp} \equiv e \int d^2\vec{b}\,(b_x^2 - b_y^2)\, \rho^{\Delta}_{T\,s_\perp}(\vec{b})$$

$$Q^{\Delta}_{\frac{3}{2}} = -Q^{\Delta}_{\frac{1}{2}} = \frac{1}{2}\left\{2\left[G_{M1}(0) - 3e_\Delta\right] + \left[G_{E2}(0) + 3e_\Delta\right]\right\}\left(\frac{e}{M_\Delta^2}\right)$$

$s_\perp = +3/2$

for spin-3/2 point particle

$G_{M1}(0) = 3e_\Delta$ and $G_{E2}(0) = -3e_\Delta$



transverse charge densities
depend only on anomalous
values of e.m. moments
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Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)

Transverse charge densities for

N -> Δ (1232), P_{11}(1440), S_{11}(1535), D_{13}(1520),...

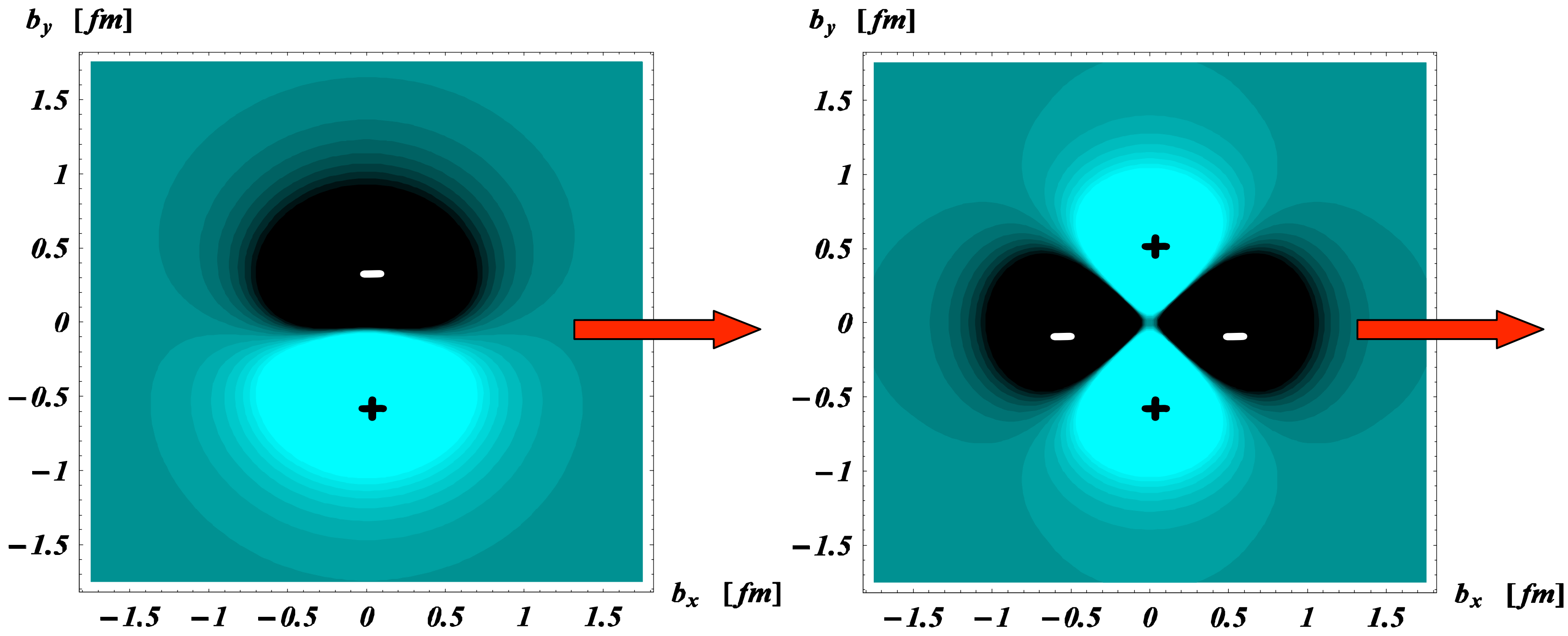
N -> Δ(1232) transition densities in transverse spin state

$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2}\,|\,J^+(0)\,|\,P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2}\rangle$$

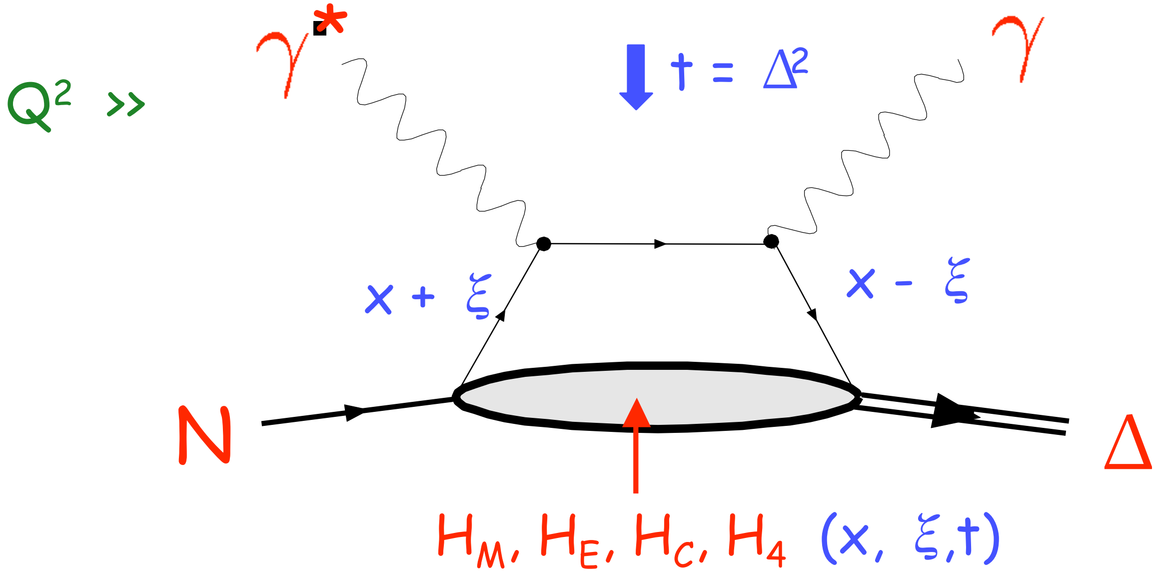
$$= \int_0^\infty \frac{dQ}{2\pi}\,\frac{Q}{2}\,\left\{ J_0(b\,Q)\, G^+_{+\frac{1}{2}\,+\frac{1}{2}} \right.$$ → monopole

$$- \sin(\phi_b - \phi_S)\, J_1(b\,Q)\left[\sqrt{3}G^+_{+\frac{3}{2}\,+\frac{1}{2}} + G^+_{+\frac{1}{2}\,-\frac{1}{2}}\right]$$ → dipole

$$\left. - \cos 2(\phi_b - \phi_S)\, J_2(b\,Q)\sqrt{3}\, G^+_{+\frac{3}{2}\,-\frac{1}{2}} \right\}$$ → quadrupole



N -> Δ DVCS and GPDs



low -t process :
-t << Q^2

$$\int_{-1}^{1} dx\, H_M(x,\xi,t) = 2\, G_M^*(t)$$

$$\int_{-1}^{1} dx\, H_E(x,\xi,t) = 2\, G_E^*(t)$$

$$\int_{-1}^{1} dx\, H_C(x,\xi,t) = 2\, G_C^*(t)$$

$$\int_{-1}^{1} dx\, H_4(x,\xi,t) = 0$$

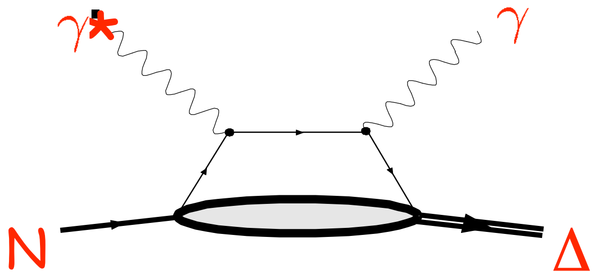
Jones-Scadron

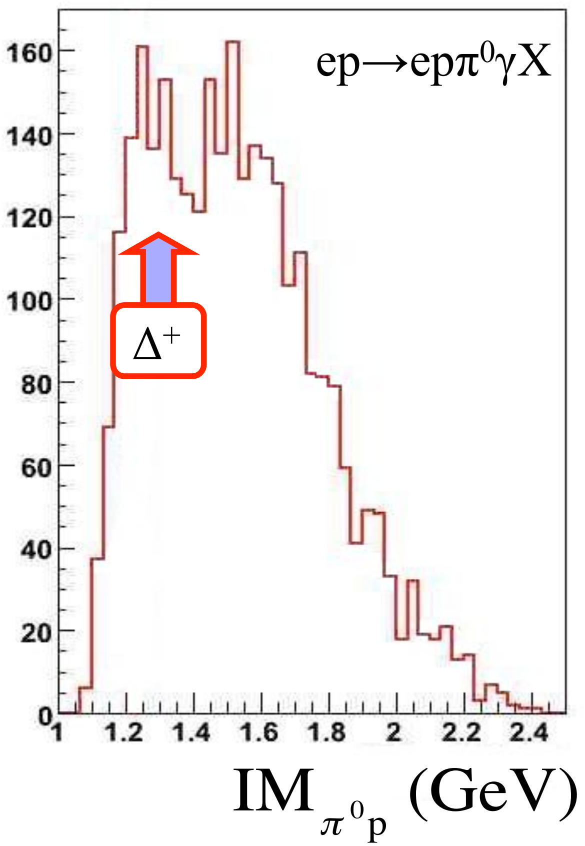
N -> Δ form factors

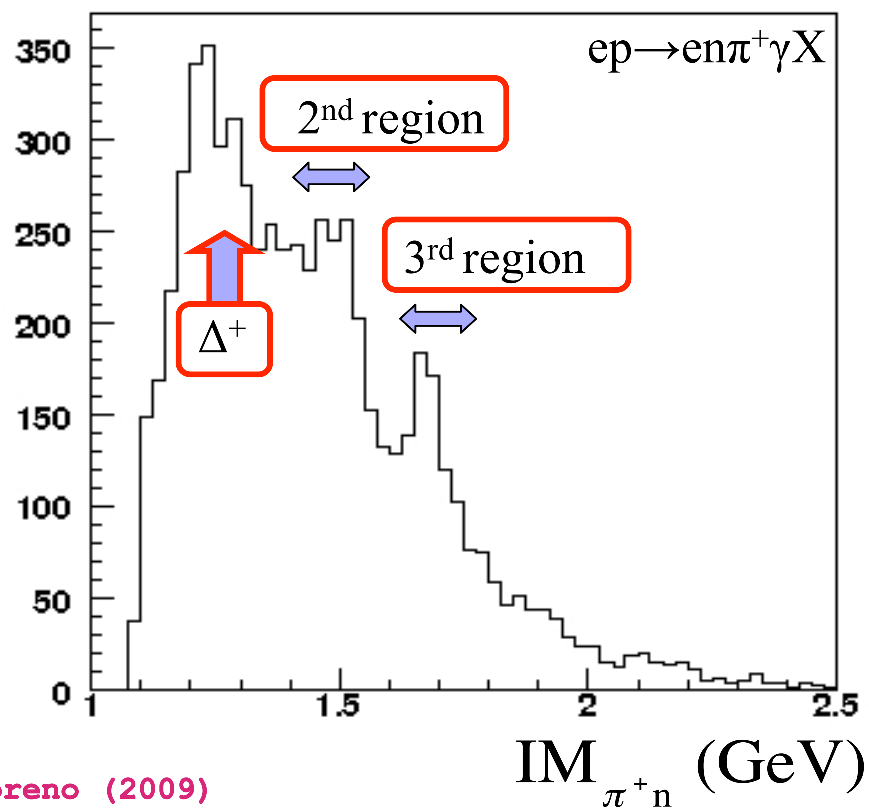
N -> Δ DVCS events in CLAS

W > 2 GeV

$Q^2 \approx 2.5\ GeV^2$







Moreno (2009)

N -> Δ magnetic dipole GPD and FF

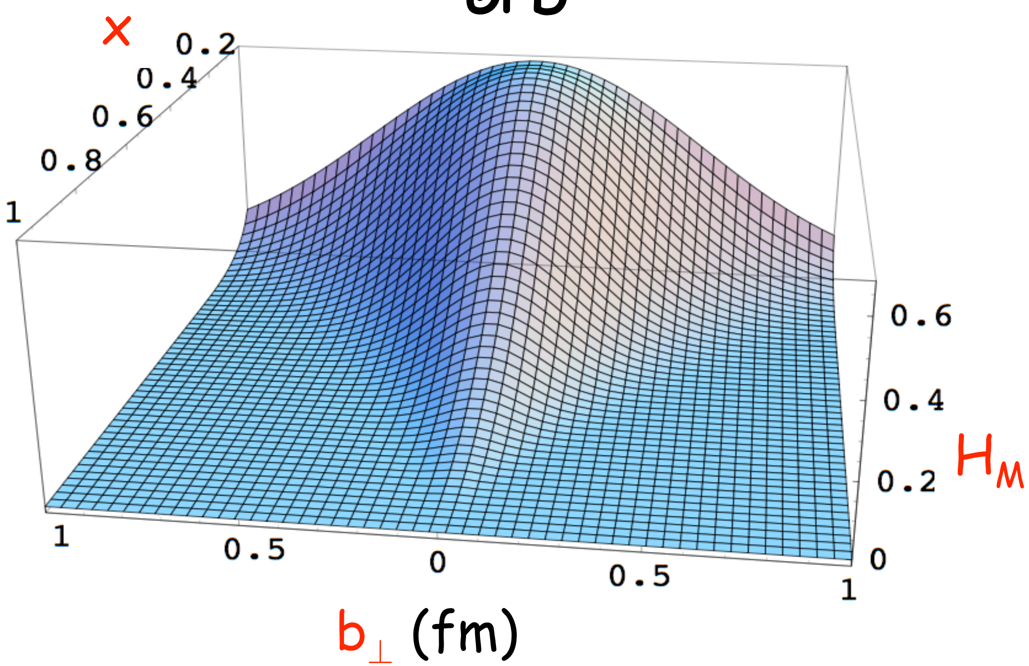
large N_c : $G^*_M(0) = \kappa_V / \sqrt{2} = 2.62$

EXP : $G^*_M(0) = 3.02$

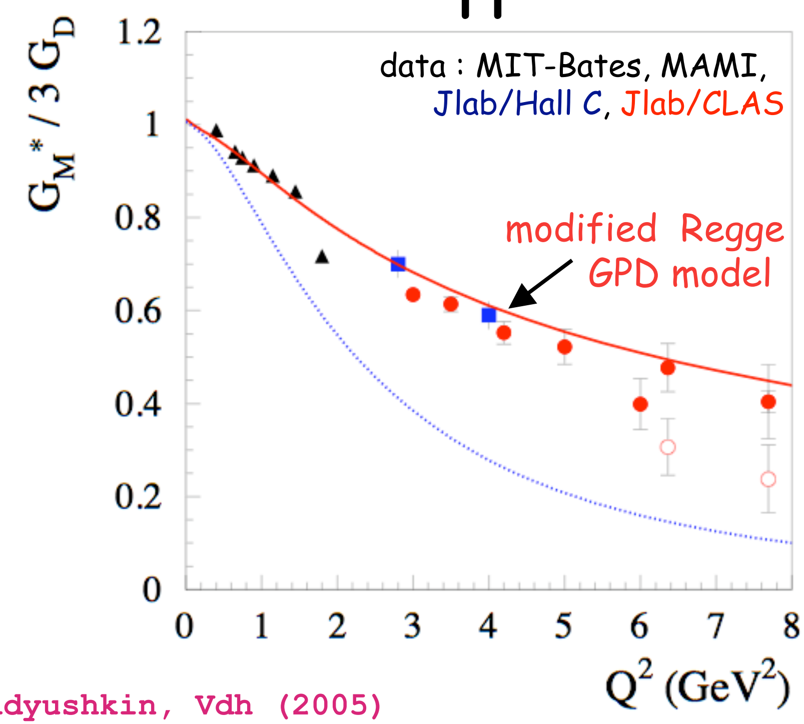
large N_c limit

$$G^*_M(t) = \frac{G^*_M(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x,\xi,t) - E^d(x,\xi,t) \right\} = \frac{G^*_M(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$

GPD

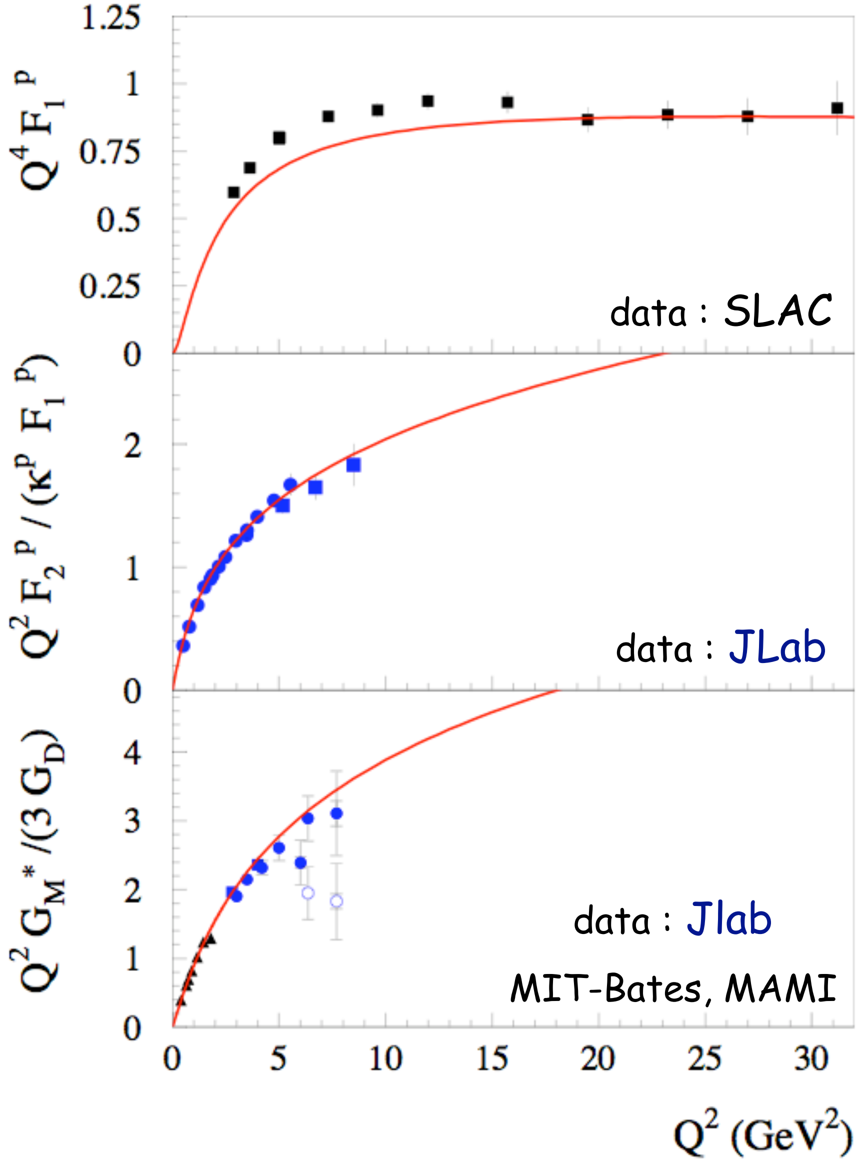


FF

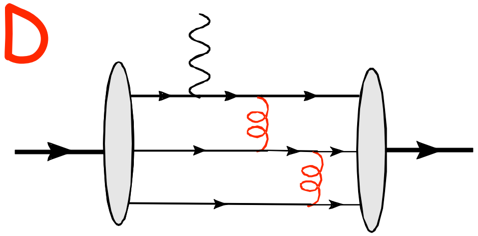


Guidal, Polyakov, Radyushkin, Vdh (2005)

scaling behavior of N and N -> Δ FF



PQCD



+ collinear quarks

$F_1{}^p \sim 1/Q^4$

$F_2{}^p / F_1{}^p \sim 1/Q^2$

$G_M{}^* \sim 1/Q^4$

GPD —— modified Regge GPD model

Guidal, Polyakov, Radyushkin, Vdh (2005)

N -> Δ : E2 and C2 FFs

➡ large N_c limit of QCD :

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2\, \frac{N_c}{N_c+3}\sqrt{\frac{N_c+5}{N_c-1}}$$

Buchmann, Hester, Lebed (2002)

EXP : r_n^2 = - 0.113 (3) fm^2

large N_c : $Q_{p\to\Delta^+}$ = - 0.080 fm^2

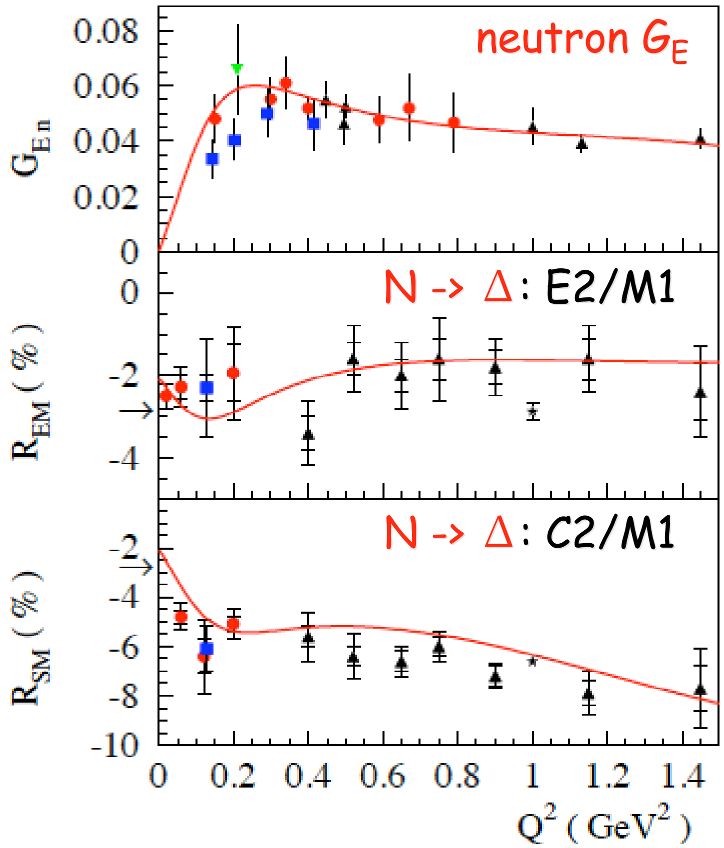
EXP : $Q_{p\to\Delta^+}$ = - 0.085 (3) fm^2

➡ finite (low) Q^2 :

$$G_E^*(Q^2) \simeq \frac{1}{\sqrt{2}}\,\frac{(M_\Delta^2 - M_N^2)}{2}\,\frac{G_E^n(Q^2)}{Q^2}$$

$$G_C^*(Q^2) \simeq \frac{4M_\Delta^2}{(M_\Delta^2 - M_N^2)}\, G_E^*(Q^2)$$

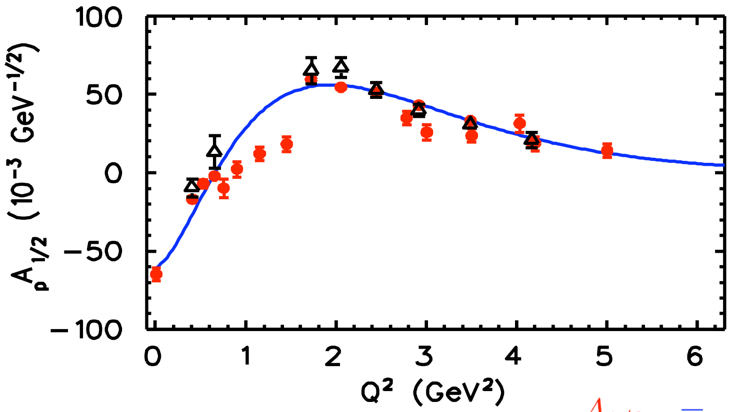
Pascalutsa, Vdh (2006)



G_{En} fit: Bradford, Bodek, Budd, Arrington (2006)

E2,C2 data: MAMI, NIKHEF, MIT-Bates, JLab

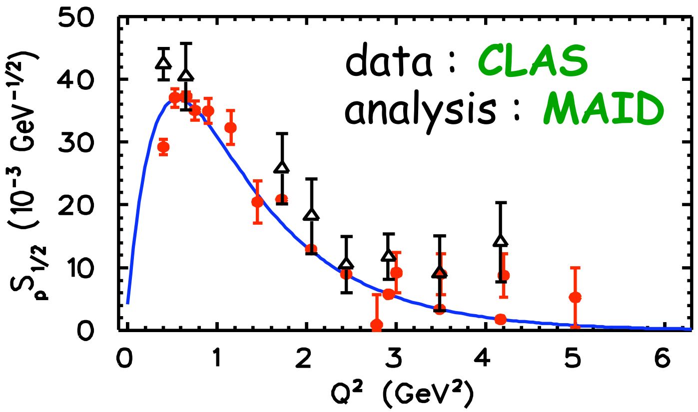
empirical transition FFs for $p \to P_{11}(1440)$ excitation



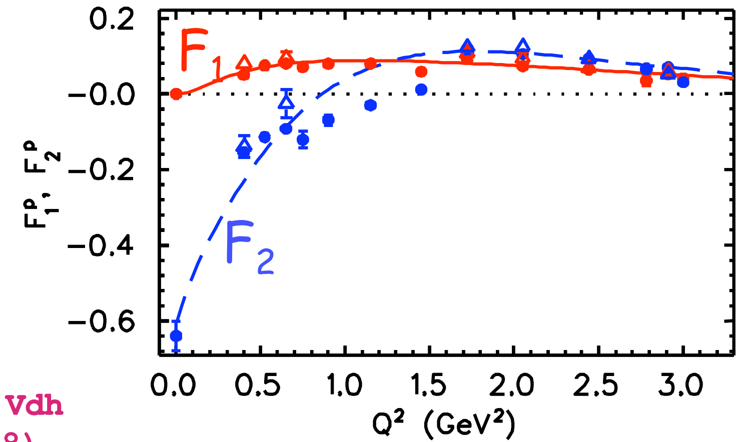
$$\langle N^*(p',\lambda')\,|\,J^\mu(0)\,|\,N(p,\lambda)\rangle = \bar{u}(p',\lambda')\left\{F_1^{NN^*}(Q^2)\left(\gamma^\mu - \gamma\cdot q\,\frac{q^\mu}{q^2}\right) + F_2^{NN^*}(Q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{(M^*+M_N)}\right\}u(p,\lambda)$$

$$A_{1/2} = e\,\frac{Q_-}{\sqrt{K}\,(4M_N M^*)^{1/2}}\left\{F_1^{NN^*} + F_2^{NN^*}\right\}$$

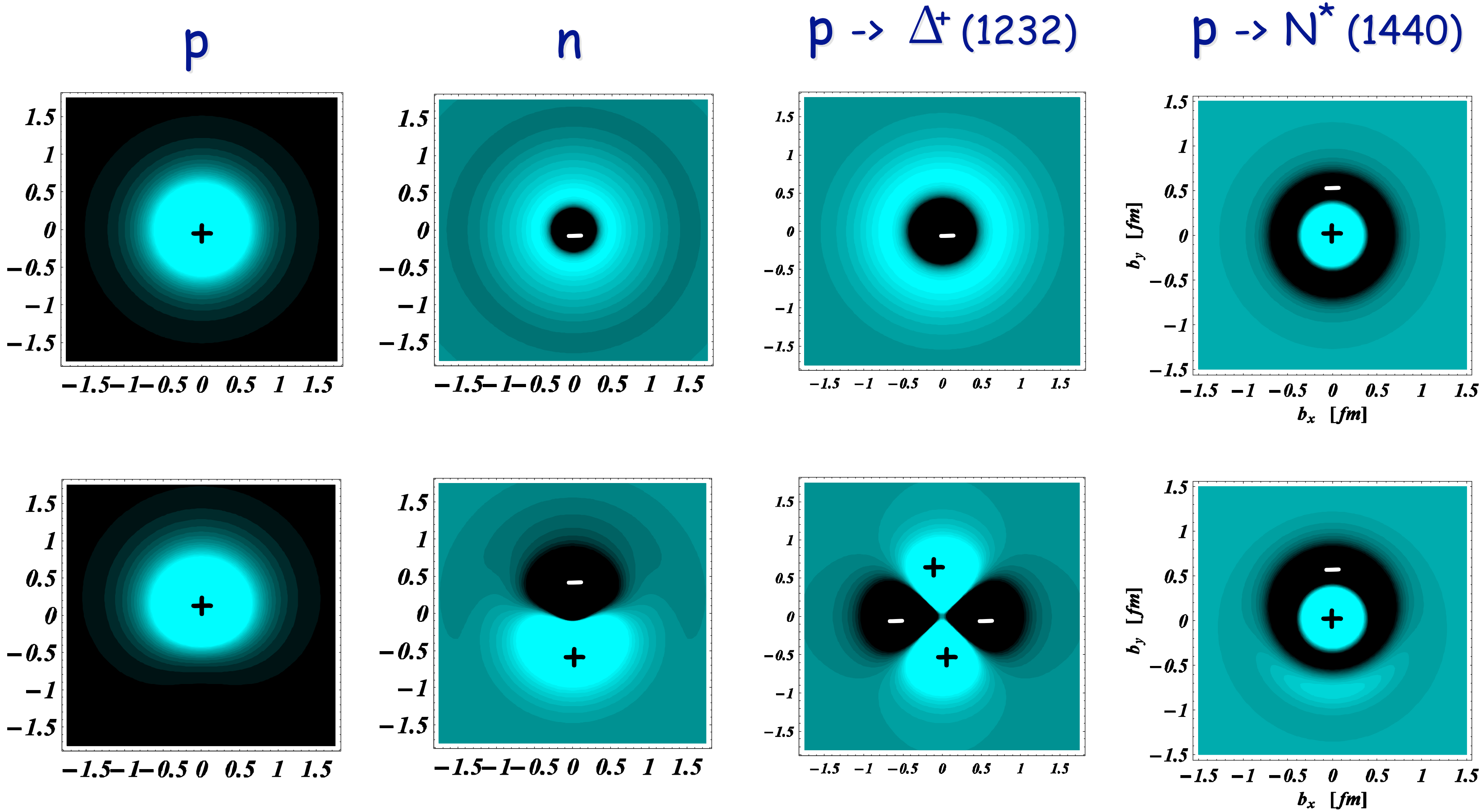
$$S_{1/2} = e\,\frac{Q_-}{\sqrt{2K}\,(4M_N M^*)^{1/2}}\left(\frac{Q_+Q_-}{2M^*}\right)\frac{(M^*+M_N)}{Q^2}\left\{F_1^{NN^*} - \frac{Q^2}{(M^*+M_N)^2}F_2^{NN^*}\right\}$$



Tiator, Vdh (2008)



empirical transverse transition densities

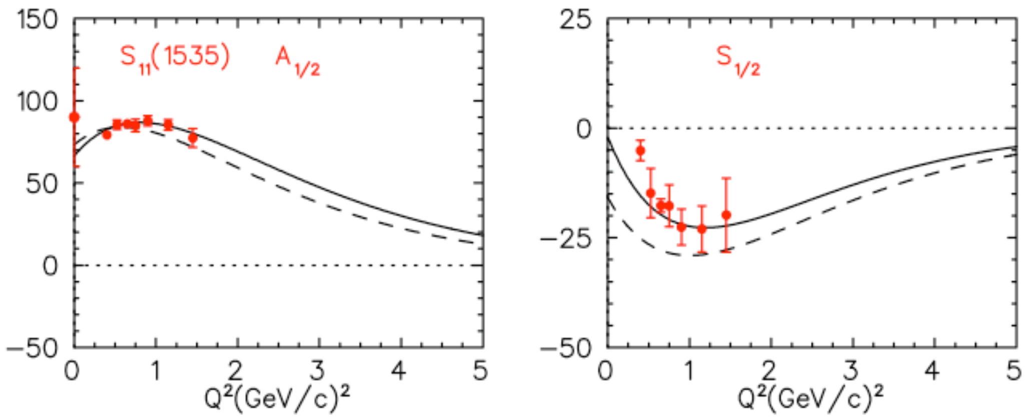


Carlson, Vdh (2007)

quadrupole pattern

Tiator, Vdh (2008)

empirical transition FFs for p -> $S_{11}(1535)$, $D_{13}(1520)$ excitations

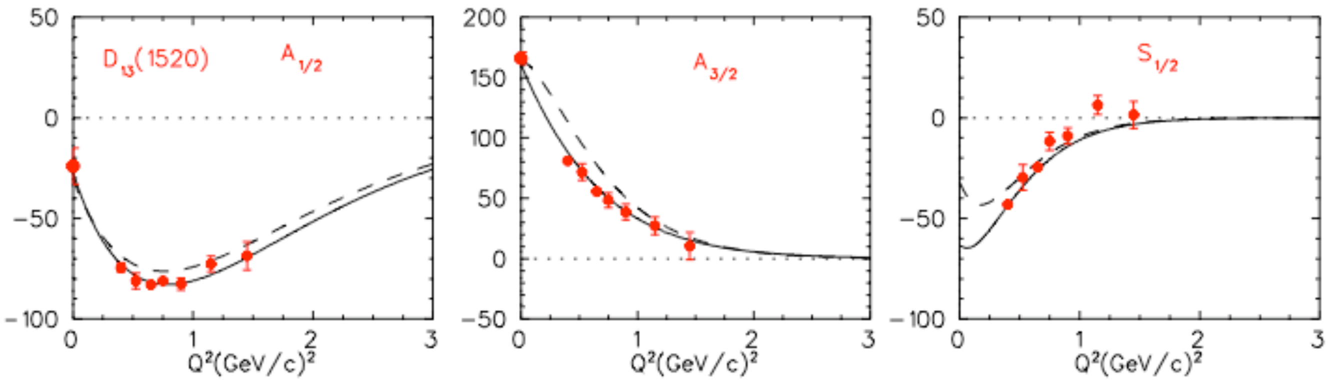


data : **CLAS**

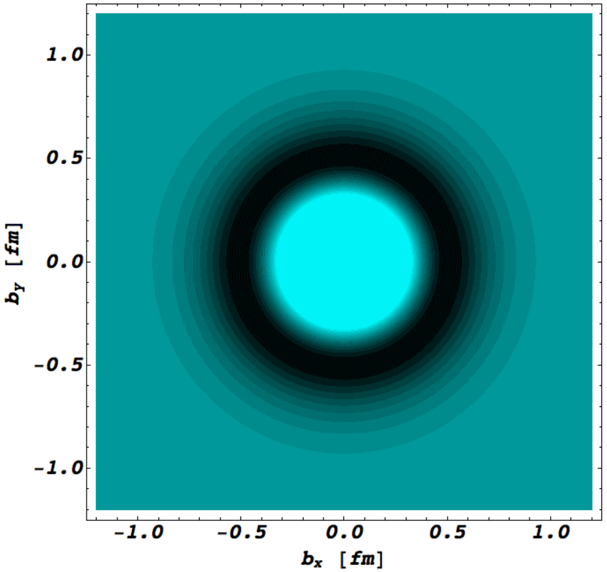
analysis : **MAID**

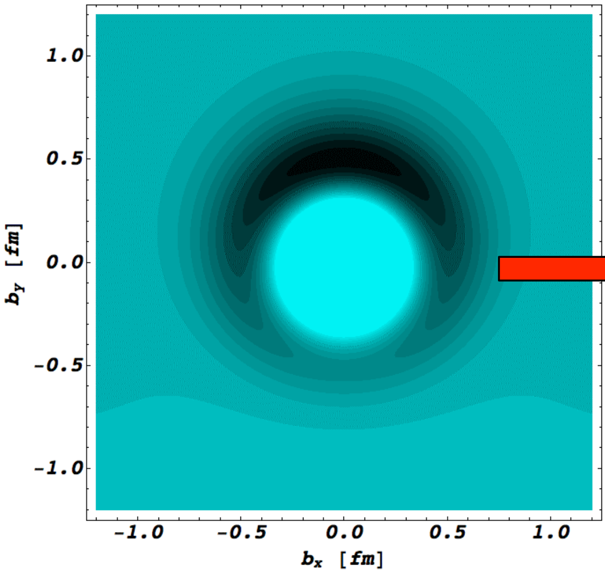
——— 2007

– – – 2003

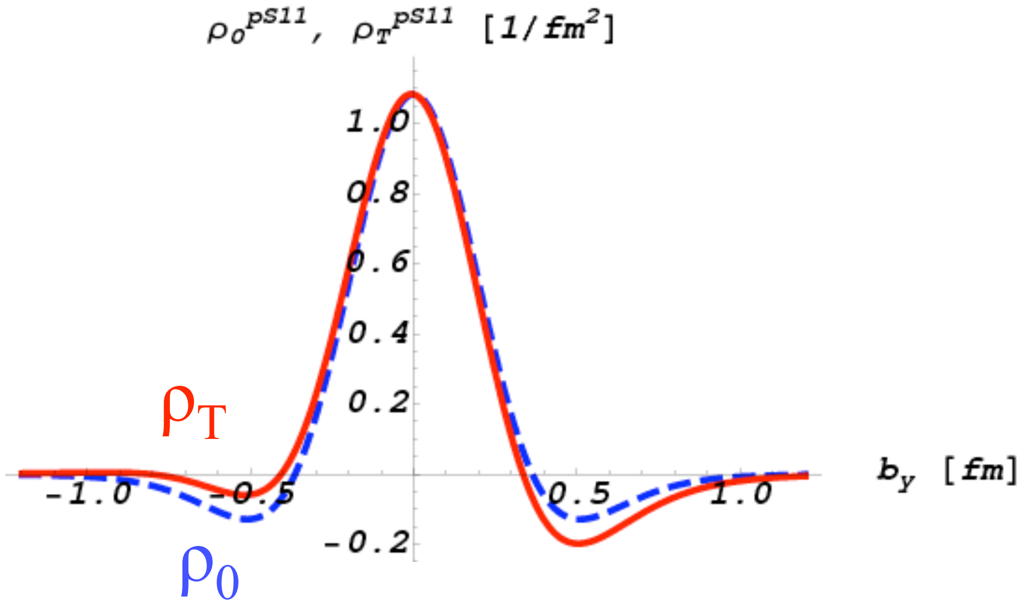


empirical transverse transition densities for p -> $S_{11}(1535)$ excitation



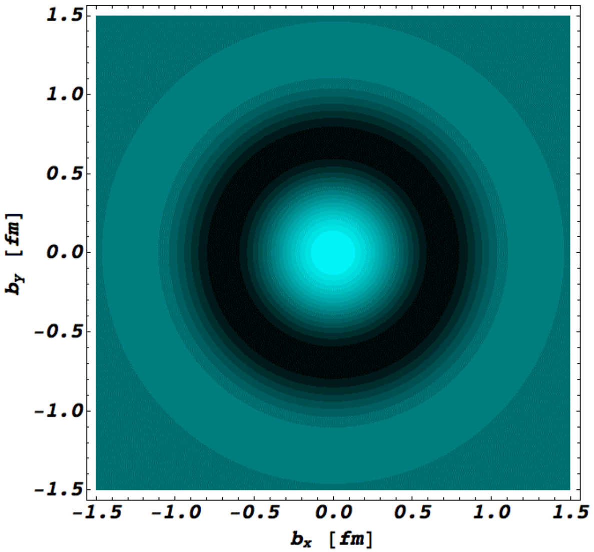


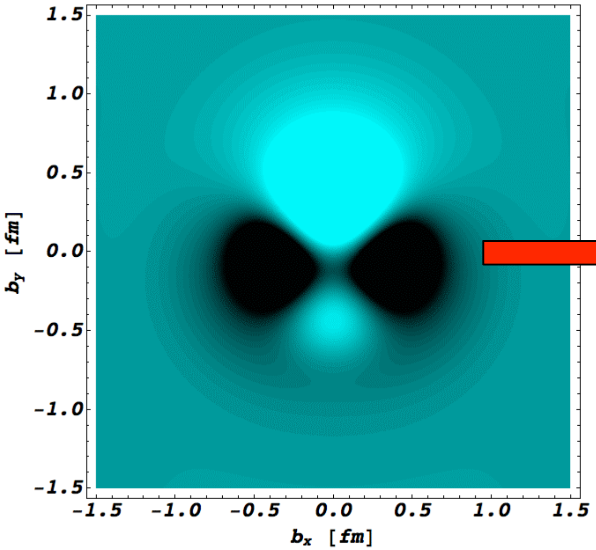
transition $s_\perp = +1/2 \rightarrow s_\perp = -1/2$



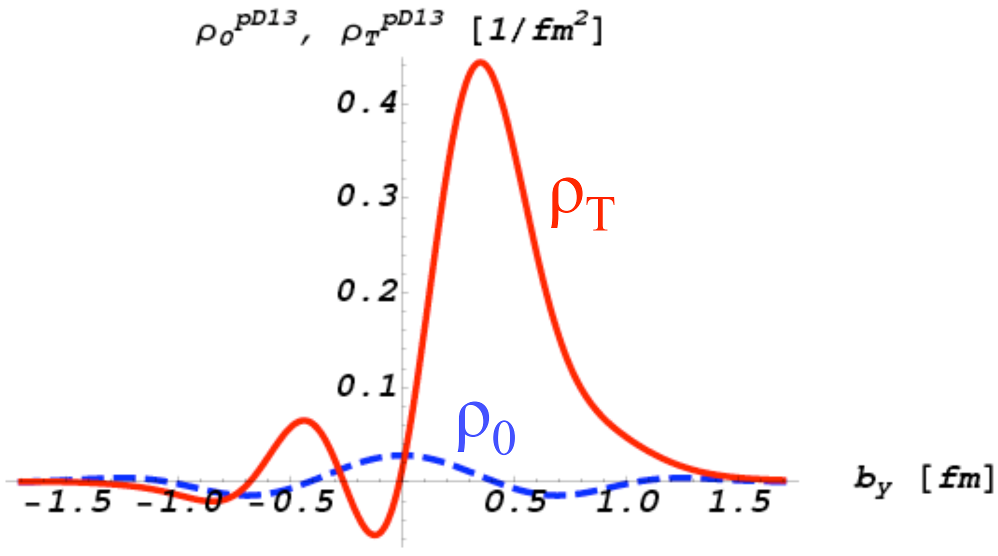
Tiator, Vdh

empirical transverse transition densities for p -> $D_{13}(1520)$ excitation





transition $s_\perp = +1/2$ -> $s_\perp = -1/2$



Summary

Light-front charge densities provide a 2D transverse imaging of hadrons

- elastic nucleon form factors : empirical transverse charge densities reveal different spatial distributions of u / d quarks
- Shape of hadrons can be understood within relativistic quantum field theory from higher e.m. moments of transverse charge densities as deviations from their "natural" values
- electromagnetic structure of the Δ(1232) resonance

  Lattice QCD results allows to access transverse charge densities, pointing to a prolate Δ deformation when viewed from the light-front
- N -> N^* transition form factors allow to map out the nucleon-resonance transition charge densities

  Δ(1232), P_{11}(1440), S_{11}(1535), D_{13}(1520),…